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Development of awareness of tolerance and mathematical reasoning skills through action research in multicultural mathematics education

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ABSTRACT

Globalization, increased ease of transportation, and conflicts in different regions have led to multicultural interactions among diverse communities. However, there is a lack of research on tolerance awareness in multicultural mathematics education. This study implemented action research to develop middle school students' awareness of tolerance and their mathematical reasoning skills in multicultural mathematics education. Researchers involved twenty-three sixth-grade students in this research. Data collection included quantitative and qualitative methods. Researchers conducted three cycles and used six action research plans in the study. The researchers used content analysis to analyze the data. The results indicated that, before the action research, students had a low awareness of tolerance development and had limited mathematical reasoning skills. However, after the action research, we found remarkable improvements regarding awareness of the development of tolerance and mathematical reasoning skills in a multicultural mathematics education context. Based on these findings, recommendations for implementing multicultural mathematics education are formulated to overcome the difficulties and challenges teachers and students experience when applying action research methods in further studies.

Keywords: mathematics education, multicultural education, flipped classroom model, tolerance, mathematical reasoning skills, multicultural mathematics education

INTRODUCTION

Many nations have recognized the importance of mathematics education and undertaken educational reforms to ensure students acquire proficient mathematical literacy (Moody, 2014; Taley & Azumbila, 2024). For this aim, researchers have been exploring innovative methods and techniques to enhance the effectiveness of mathematics instruction (van Alten et al., 2019). Integrating technology with mathematics education is one of the most innovative methods and techniques. One of the most notable innovative examples is the flipped classroom model (FCM). This approach facilitates convenient access to necessary materials, promotes efficient use of time, diversifies learning through exposure to various stimuli, and develops high-level skills through student-centered activities under teacher guidance (Al-Said et al., 2023; Amstelveen, 2018; Li et al., 2022). Parallel to technological developments, globalization and conflicts in different regions have led to multicultural societies emerging worldwide. Multicultural education has been implemented to provide equitable educational opportunities for students from diverse racial, ethnic, and social backgrounds. Thus, education has adapted to accommodate multiculturalism.

Although multiculturalism has made significant progress in countries like Canada and Australia (Arslan, 2016), scholars in developing countries have not promoted multicultural education in teaching subject-specific topics to students (Arslan, 2016; Cırık, 2008). Nevertheless, scholars reported that mathematics instruction has not adequately addressed value education efforts (Arslan, 2016). Moreover, there has been a noticeable scarcity of studies addressing multiculturalism in the context of mathematics education in middle school. The increasing use of tablets and cell phones among students and their low interest and achievement in mathematics due to traditional teaching necessities the use of FCM in educational research. Therefore, it is essential to integrate the FCM and multicultural education in mathematics education. The use of mathematics instruction in a multicultural environment allows the integration of FCM, which can be tailored to different cultural backgrounds, enabling a more culturally sensitive teaching approach to mathematics lessons. Thus, we aimed to assess the development of flipped classroom learning with multicultural education by developing mathematical reasoning skills and tolerance using the action research method.

In this research, researchers chose to use action research due to several reasons. Firstly, there has been a recurring problem over the past nine years concerning the struggle of international students to adapt to the social environment in researcher's schools. For example, Turkish students were reluctant to form friendships with their foreign peers. This lack of close relationships between Turkish and international students prevented international students from fully embracing cultural differences. As a result, Turkish students were using some terms like "Syrian," "Afghan," and "Iraqi" instead of addressing foreign students by their names due to the absence of an established cultural connection. This situation was evidenced by Sarıer's (2020) findings. The results of previous research and one of the authors of this paper observed instances where Turkish students displayed intolerance towards the cultural differences of international students. This lack of tolerance made international students feel disconnected, negatively impacting their social and community integration. Secondly, their performance in math courses was low s due to social integration and poor relations with Turkish students, particularly regarding tolerance for cultural differences. Another problem was that students had difficulty solving mathematical problems. As an experienced mathematics teacher, the researchers observed that technological tools used in the classroom better capture students' attention and lead to more active participation. Students were observed to use technology after school for research related to the course or topics they found difficult in mathematics class. Consequently, action research was chosen to overcome these problems. To address these problems in an educational context, this action research aimed to address multiculturalism in the school setting and improve students' mathematical reasoning skills and their awareness of tolerance through FCM.

LITERATURE REVIEW

Multicultural Mathematics Education

Scholars have conducted less research on multicultural mathematics education than on the FCM. A few studies have been undertaken to understand the development of multicultural instruction for students and teachers. For example, Greer et al. (2009) found that incorporating multicultural perspectives into math instruction was associated with improved problem-solving skills and can lead to better academic performance. For instance, Battey and Leyva (2016) found that students who engaged in math content that reflected their cultural background and experiences had more positive attitudes toward mathematics. In another research, Yao (2016) examined the development of culture-based math modules and guided the development of culture-based math modules to make math more relevant and meaningful for Indigenous students. Mendrofa et al. (2022) emphasized the role of multicultural education in forming students' attitudes and values through mathematics education.

Smith et al. (2009) explored that schools with robust multicultural programs significantly enhanced reading and math scores for African American and Hispanic students compared to schools without similar programs. Leonard et al. (2014) found that students who chose a multiculturally based mathematics textbook were more academically successful and acquired higher cultural competence and critical thinking skills than those who did not. Parallelly, research by Türkkan (2017) demonstrated that the development of teaching mathematics using action research that integrates socio-mathematical topics helped to increase students' awareness of social justice and equity values, and their problem-solving skills significantly after the action research. The results of the research studies provide educators and teachers with a solid base for understanding the outcomes of multicultural mathematics education in mathematics teaching. However, to our knowledge, none of these studies have merged flipped learning and multicultural mathematics education in the context of middle school students and examined the development of mathematical reasoning skills and tolerance.

Flipped Classroom Model

Flipped learning reverses the traditional teaching model by delivering instructional content outside of class time, often through video lectures, and moving "homework" into the classroom to allow for active learning strategies during class time. - scholars have conducted much more research on flipped classroom teaching models to understand the development of the FCM on student achievement, attitudes, and engagement in mathematics. For example, Abeysekera and Dawson (2015) reported that students in flipped math courses were likelier to participate in class and exhibit higher motivation to learn. Zainuddin and Halili (2016) found that flipped learning environments promote active learning and peer collaboration, contributing to a more positive attitude toward mathematics. A meta-analysis by van Alten et al. (2019) found that flipped forms of instruction generally lead to better academic achievement than traditional forms of instruction. Researchers indicated that the best way to integrate technology into education was to apply the FCM (Oliveira & Pombo, 2017); math skills increased significantly with the FCM (Wei et al., 2020). FCM decreased students' math anxiety, increased their ability to concentrate, and increased their self-efficacy as mathematics students (Butler James, 2020).

Tolerance

Tolerance means respecting, understanding, and accepting other thoughts, beliefs, cultures, and lifestyles. In today's world, diversity has led to multicultural societies where different cultures, languages, and traditions live together. Hence, it is crucial to improve tolerance views in multicultural societies. Only one research on tolerance and mathematics education was conducted at the middle school level. For example, Suyitno et al. (2019) found ways to integrate character values into learning, especially in mathematics lessons. The results from these studies explicitly show the need for more research on tolerance in multicultural mathematics education.

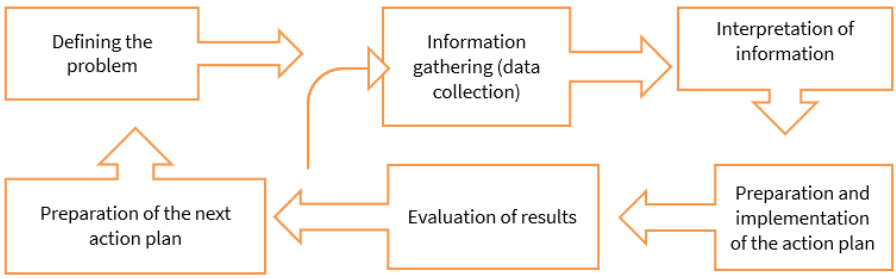

Figure 1. Action research process-1 (Source: Authors' own elaboration)

Mathematical Reasoning Skills

Mathematical reasoning skills are essential components that hold other elements together in performing operations, synthesizing concepts, and solving problems (Hasanah et al., 2019) to plan a solution as the first step when confronted with a problem and minimize and apply solution strategies to solve problems. Findings from previous empirical studies (Hasanah et al., 2019; Mumu & Tanujaya, 2019; Pratiwi & Pujiastuti, 2020) provide valuable insights into the various factors influencing mathematical reasoning skills, including deductive reasoning, educational interventions, and the learning environment. In particular, no previous studies have examined the impact of flipped learning and multicultural mathematics education on students' mathematical reasoning skills.

Rationale and Importance

As Karacabey et al. (2019) suggested that there is an always need for further research on how teaching methods and programs can be changed and adapted in line with the principles of multicultural education. From this point of view, flipped learning aligns well with the goals of multicultural mathematics education, which aims to respect and integrate different cultural perspectives into the teaching and learning processes. Flipped learning materials can be designed to include examples, problems, and contexts that are culturally relevant and inclusive, making math more understandable and engaging for students from diverse cultural backgrounds. The results of previous research have shown that most of the participants in earlier studies are teachers or prospective teachers (e.g., Ivenicki, 2021; Janakiraman et al., 2019; Leonard et al., 2014). These studies mainly examined the development of courses prepared with a multicultural approach to increase academic achievement. Because of this reason, the number of studies on mathematics education and multicultural education at middle school level is very limited. The lack of research on mathematics and multicultural education at the middle school level addresses a research gap in literature.

For this reason, the present research aims to add a new perspective to the researchers' and teachers' existing knowledge and improve students' mathematical reasoning skills using the action research method. In addition, this research aims to fill out the existing research gap. Thus, the results of this research will contribute to the integration processes of foreign students using flipped learning in mathematics education in the classroom and their social lives.

Purpose

This research assessed the development of flipped classroom learning with multicultural education in terms of mathematical reasoning skills and tolerance development through action research. To achieve these objectives, the following research questions are explored:

1. What was the initial level of awareness among sixth-grade middle school students regarding the development of tolerance and their mathematical reasoning skills before action research?
2. What is the level of students' awareness regarding the development of tolerance and mathematical reasoning skills after implementing mathematics instruction through FCM, developed based on multicultural education principles after action research?

METHOD

This research used action research. Action research is a methodology that involves collecting and analyzing qualitative and quantitative data to gain insight into and address issues encountered by educators or teachers (Beyhan, 2013). It constitutes a structured research process that is meticulously planned and organized, undertaken by educators facing real-world challenges within an authentic school environment to enhance the quality of education and training (Johnson, 2003). **Figure 1** shows the stages of the action research process.

As depicted in **Figure 1**, the initial step of this study involved defining the problem. Researchers conducted a comprehensive literature review as the first step to determine the problem. Researchers identified the research topic based on the insights gained from the literature review, and the problem was formulated. To define the problem, a pilot study was undertaken to collect relevant data. In this pilot study, assessment forms for determining the development of mathematical reasoning and gauging awareness of the development of tolerance were administered to sixth-grade students based on the collected data, researchers analyzed the need for this present research and meticulously developed action plans and implemented them. Following the execution of each action plan, a new action plan was designed, and the research cycle was determined.

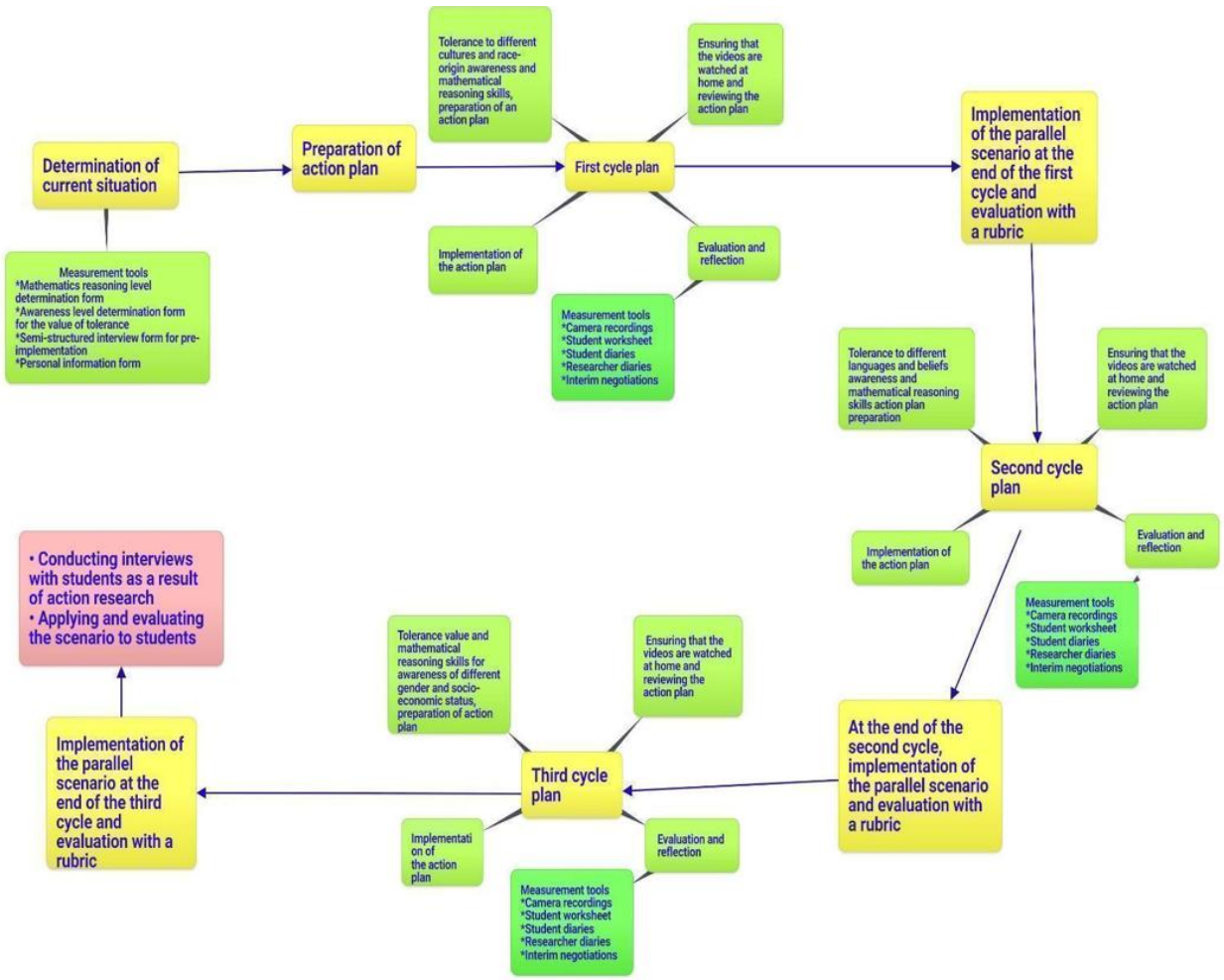

Figure 2. Action research process-2 (Source: Authors' own elaboration)

Figure 2 shows the action research process used in this research. As indicated in **Figure 2**, researchers developed action plans and formulated a strategy that involved two actions within each cycle. Within each cycle, we designed and executed activities targeting two sub-dimensions of multiculturalism and two sub-dimensions of mathematical reasoning skills.

Participants

This study encompassed twenty-four sixth-grade students enrolled at a public middle school in Nigde Province during the 2021-2022 academic year. The participants were selected using a purposeful sampling technique. The following established characteristics guided the process of determining the study group for this research:

- Enrollment in the sixth grade of middle school during the 2021/2022 academic year
- Inclusion in a heterogeneous class characterized by cultural, linguistic, and ethnic diversity due to the presence of immigrant students from countries such as Syria, Palestine, and Iraq
- Inclusion of students with diverse academic achievement levels within the classroom

Some students migrated from Palestine, Syria, and Iraq. The school had 350 students.

Data Collection Tools

The data collection tools were a personal information form, an awareness form of tolerance, a test for mathematical reasoning skills, interviews, a student self-evaluation form, researcher and student diaries, and scenarios. **Appendix A** provides more detailed information about the data collection tools.

Data Collection

The action research process began in March 2022 and ended in April 2022. The six sub-dimensions of multiculturalism and the seven sub-dimensions of mathematical reasoning skills were planned so that two sub-dimensions were included in the timetable for each subject. Each week, students were taught one sub-dimension on multiculturalism and one sub-dimension on mathematical reasoning skills through classroom activities prepared along with mathematical topics. The researcher created a diary by noting the problems and salient parts throughout the study. At the end of each cycle, self-assessment forms were completed so that students could indicate the areas they considered sufficient or insufficient in the course. They were also asked to complete student diaries to describe positive and negative events about the course and notable incidents during the course.

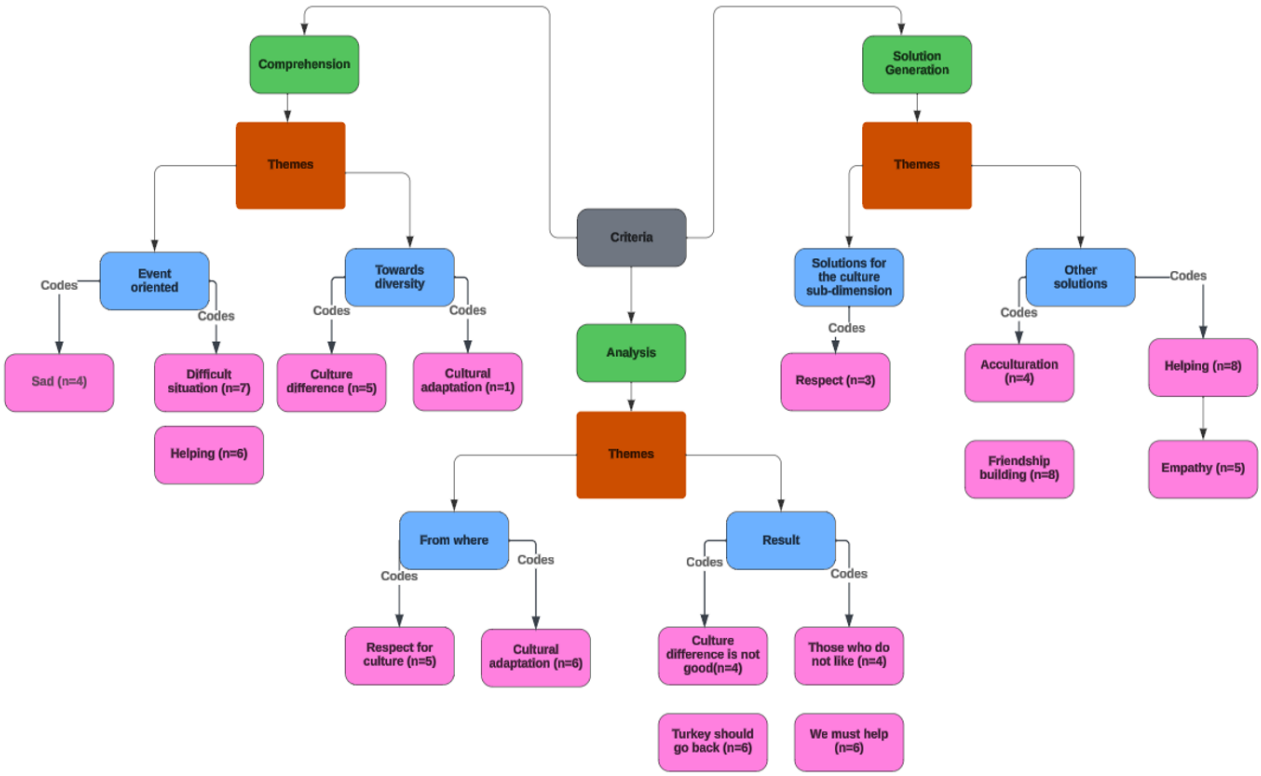

Figure 3. Codes and themes about tolerance before the action research (Source: Authors' own elaboration)

The action research process was prepared in three cycles, and each cycle included one theme from the unitized year plan, two sub-dimensions from multiculturalism, and two sub-dimensions from the mathematical reasoning skills (3 dimensions in the second cycle). At the end of each cycle, data collection instruments were presented to the participants. The videos containing the course topic, the Multiculturalism sub-dimension, and the Mathematical Reasoning sub-dimension, which the students were asked to watch at home before the cycle began, were provided by the researcher through the designated channels. In addition, the researcher monitored whether the students watched the videos before the start of the class, and the researcher solved the problems that the students had with them.

Data Analysis

The data extracted from the scenarios were assessed using rubric, while the level determination forms underwent content analysis. For the rubric, students' answers were scored. Researchers analyzed students' answers to open-ended questions by using rubric. Students' diaries were analyzed using the content analysis method. One researcher analyzed to analyze the interviews and scenarios. The second researcher analyzed a 10% ratio of the scenarios and interviews to calculate reliability. The reliability between coders was determined to be 91% for the interviews conducted before the action research and 93% for those conducted after the action research (Miles & Huberman, 2015). Furthermore, according to the Kendall tau-b (τb) correlation analysis, there were significant positive relationships between the scores of the two researchers ($\tau b = .868$, $p < .01$; $\tau b = .912$, $p < .01$; and $\tau b = .868$, $p < .01$, respectively, see **Appendix A**.

RESULTS

Before the Action Research

Awareness about tolerance

The findings about students' awareness of multiculturalism before the action research are presented in **Figure 3**. The additional findings are presented in **Appendix A**. The findings revealed that the students' tolerance awareness development towards multiculturalism was low before the actions research. Students generally could not respond to tolerance in the interviews conducted before the research. These results showed that students lacked awareness about the tolerance concept in comprehension, analysis, and solution-generation levels.

Figure 3 shows that students generally recognized a negative situation in the scenario used but had difficulty defining it. In general, students could not scientifically express that the problem that occurred at the level of comprehension, analysis, and finding a solution was due to cultural differences. Instead of describing it scientifically, they were interested in the event. They expressed short expressions such as sad, difficult situations, and the need for help due to this situation. At the comprehension level, the most frequently repeated code was "difficult situation," at the analysis level, it was "we must help," and at the solution-finding level, it was "help." In addition, while the students generally used expressions such as "difficult situation, helping, "and

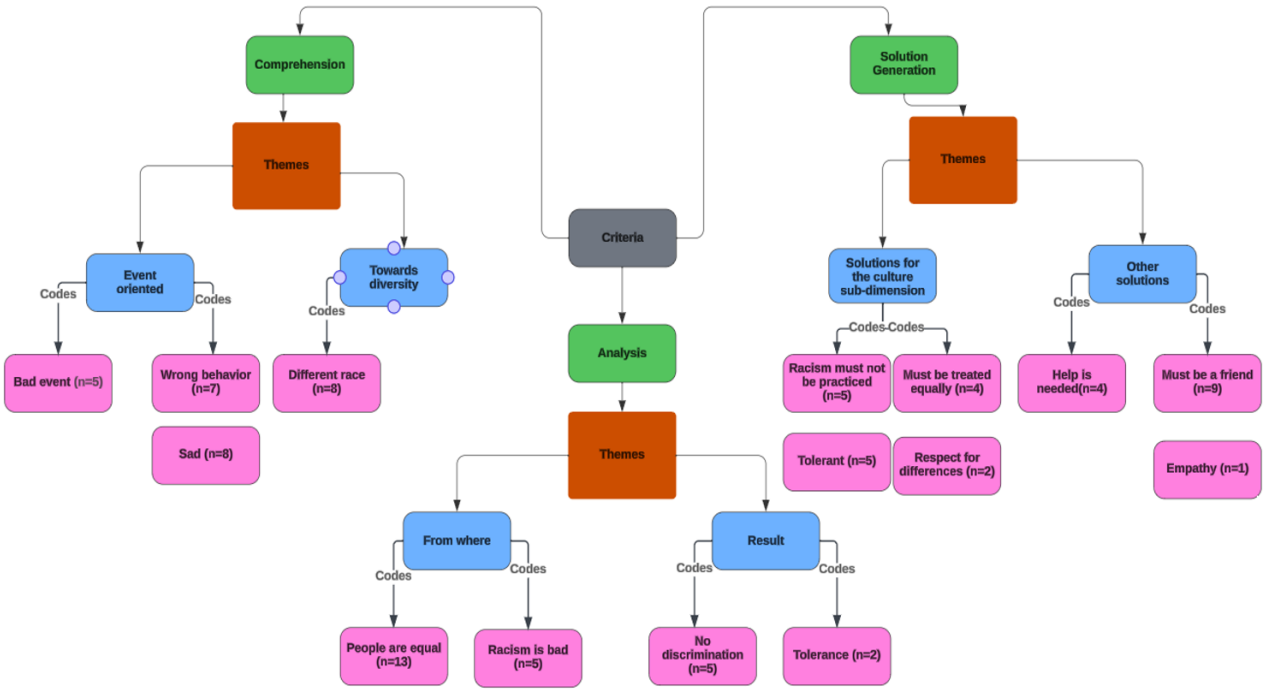

Figure 4. Codes and themes about tolerance awareness towards race-origin difference before the action research (Source: Authors' own elaboration)

"sad" about the case in the scenario in the comprehension category, only five students answered "cultural difference" about the difference that is the main reason for this event. Some quotations for the comprehension dimension are given below:

> "The situation experienced by the protagonist in the scenario is sad" (Student 9 [S9]).

> "It should be met with understanding and help him/her" (S20).

The quotations above show that the students generally described the case given themselves in scenarios as a sad and difficult situation. Few students who gave reasons made a connection with culture as a reason. Examples of quotations from the interviews in the analysis dimension are given, as follows:

> "People from other cultures may be foreigners due to cultural differences, so we should be friends with them" (S15).

> "I think people from different cultures should not be approached with prejudice" (S17).

Regarding finding a solution to the cases, the most common codes in generating solutions were "helping and making friends." Some quotations from the interviews in the dimension of generating solutions are, as follows:

> "We should empathize with the protagonist in the scenario and make friends with him so that Michael will not feel lonely" (S7).

> "People can take care of Michael, ask him how different cultures are, and integrate with him" (S17).

These findings show that the participants' answers were generally formed for other solutions instead of solutions for the sub-dimension.

Race-origin differences: Figure 4 shows that students realized a negative situation but could not understand that the sub-dimension of race origin caused this situation. While students were generally interested in the event's cause at the analysis stage, they did not express the consequences in their responses.

Findings demonstrate that students generally used expressions such as "bad event, wrong behavior, sad" about the event in the comprehension sub-dimension. Only a few participants answered "different race" about the differences. Sample quotations are given below.

> "His friends discriminated against and excluded him, so I feel sorry for him and want to help him" (S15).

> "It is a very difficult situation; the hero in the scenario was subjected to discrimination" (S3).

In the analysis dimension, the reason for the incident was read more in terms of people being equal. Quotations from the interviews are, as follows:

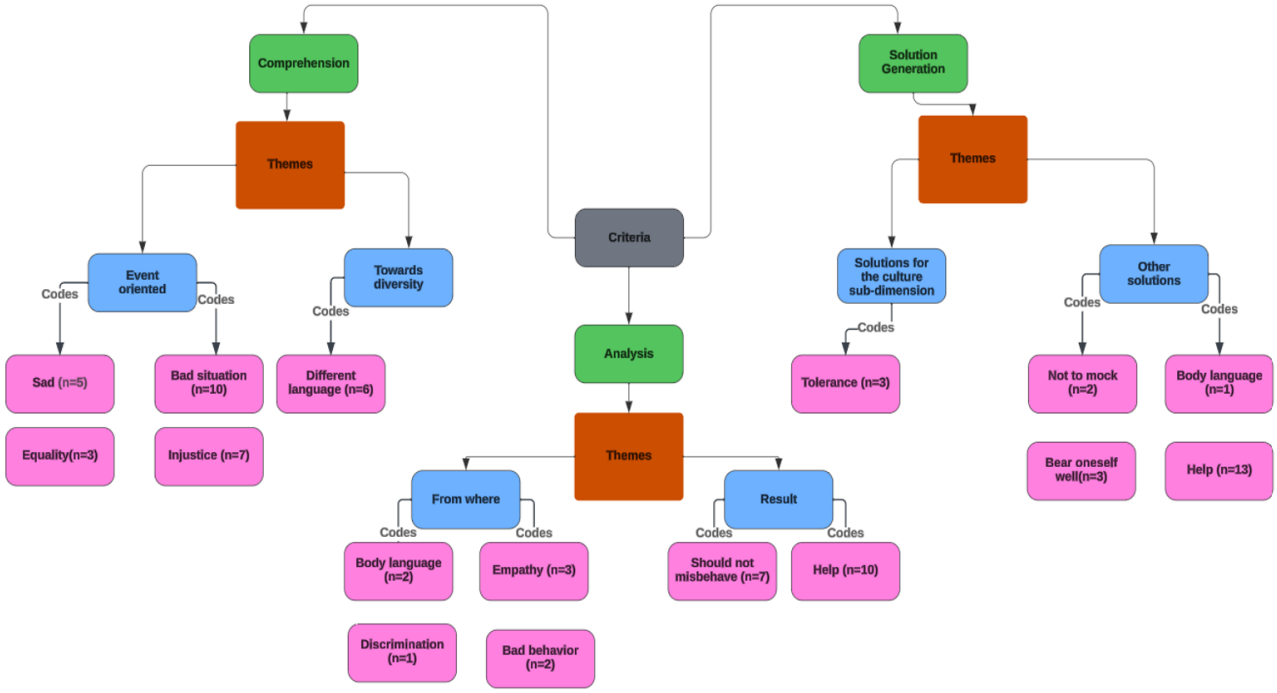

Figure 5. Codes and themes about tolerance awareness towards language differences before the action research (Source: Authors' own elaboration)

> "I do not think we should be excluded because we are all human beings, and God created us in different ways" (S16).

> "People of different colors should not be made fun of" (S21).

The most common codes in generating solutions were found to be "should be friends, should not be racist and tolerant." Quotations regarding generating solutions are, as follows:

> "I think they should sit next to the protagonist in the scenario and chat with him/her" (S18).

> "They should respect him/her and be friends with him/her" (S21).

These findings show that the participants generally tried to define the problems they experienced and expressed that they were disturbed by the negative situation. Thus, we can conclude that they could not make sufficient scientific definitions.

Language differences: Figure 5 shows findings about language differences. Students mostly formed sentences about the result while analyzing. The number of students who made sentences about the cause remained very limited. They expressed their thoughts about language differences using some words such as "bad situation" at the comprehension level. Also, they used the word "help" at the analysis and solution generation levels. The most recurring codes are "bad situation, injustice." One sample quotation in the comprehension dimension is as, follows:

> "The hero in the scenario had a lot of difficulties because he does not know the language of the place and he does not know the places at all, and there is a possibility of getting lost" (S17).

Regarding the sub-dimension of generating solutions, we found the students' responses under the solution theme. One quotation is, as follows:

> "They could give the hero in the scenario a dictionary as a gift, take care of the hero in the scenario, and buy the hero a meal" (S16).

In summary, students considered language differences. However, they could not make scientific definitions of language differences.

Differences in beliefs: Figure 6 presents findings about belief differences. Our findings revealed that students could comprehend that the root of the problem was different. Still, they could not express that different beliefs caused the problem.

At the comprehension stage, while they generally read the problem with codes such as different countries and people, the code of different beliefs was not included. At the solution generation stage, the most frequently used code at the comprehension level was "different faiths" at the analysis level.

Furthermore, we found that the students responded with "different country," "different person," and "different belief" under the theme of diversity at the comprehension level. Examples from students' statements in the comprehension dimension are, as follows:

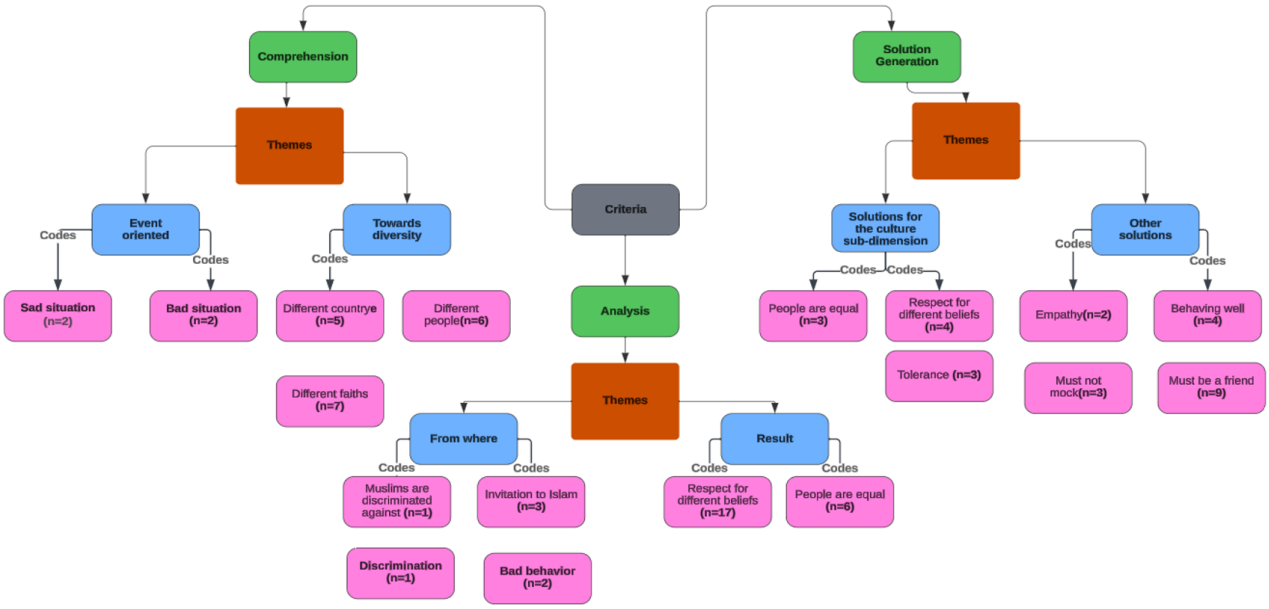

Figure 6. Codes and themes about tolerance development to difference of belief before the action research (Source: Authors' own elaboration)

"No matter what the person is, whether Muslim or Christian, discrimination should never be made" (S6).

"They treated the protagonist in the scenario badly, and they should treat people with different cultures, respectfully and well" (S8).

The most repeated codes in the analysis are "respect for different beliefs; people are equal." Sample quotations are, as follows:

"I think it would be nice if people learn each other's religions and get to know each other's cultures; it would be very nice" (S8).

"We should respect their religions and not say bad things about their religions" (S21).

The most recurring codes in the sub-dimension of generating solutions are "respect for different beliefs, should be friends."

Regarding generating solutions, they generally confused the difference in belief with the difference in culture, analyze the problem experienced accordingly, and offer solutions. The following quotations support this finding.

"If he/she does not make fun of them if he/she does not make fun of their religion, there will be no problem" (S21).

"They should respect that person and that person's religion, and they should be friends and buddies with him/her" (S18).

Gender differences: Figure 7 shows the responses regarding gender differences. The findings show that students generally stated that the problem was caused by inequality. However, they could not clearly express that the problem was caused by gender inequality. At the analysis stage, the code "women and men are equal" was found in only five answers, while the code "non-discrimination" was mainly obtained from the students' answers. This finding reveals that students have deficiencies in the concept of gender inequality.

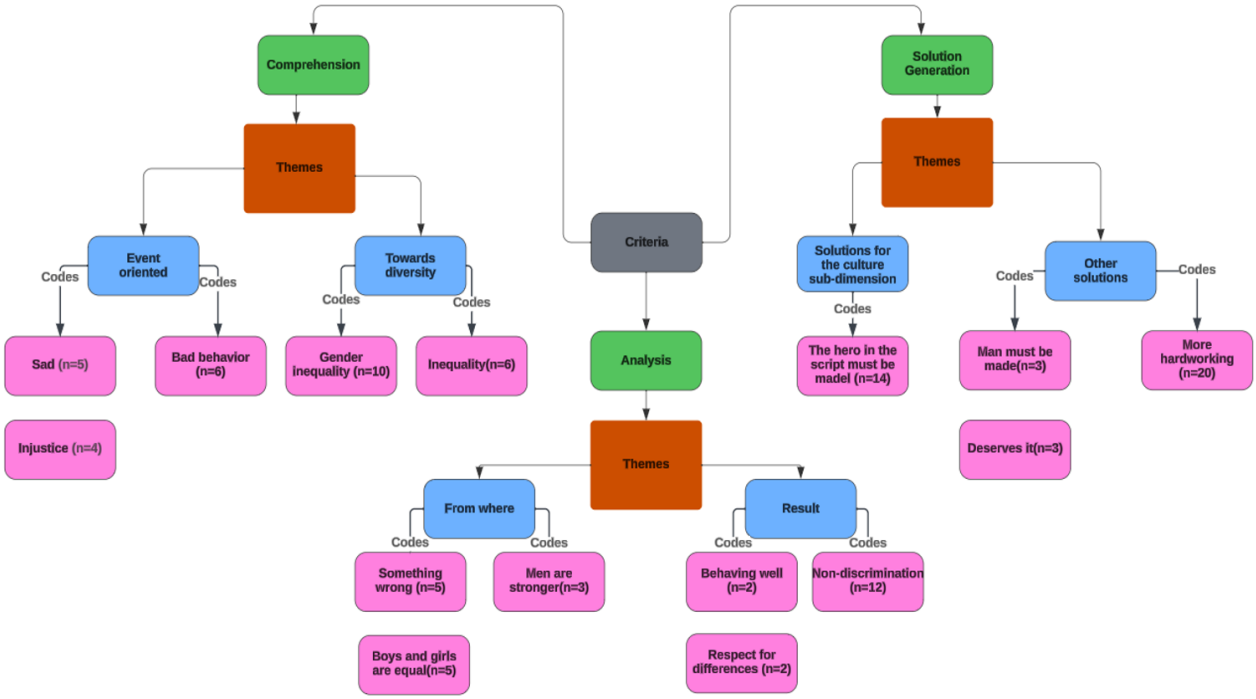

Figure 7. Codes and themes about tolerance awareness of gender difference before the action research (Source: Authors' own elaboration)

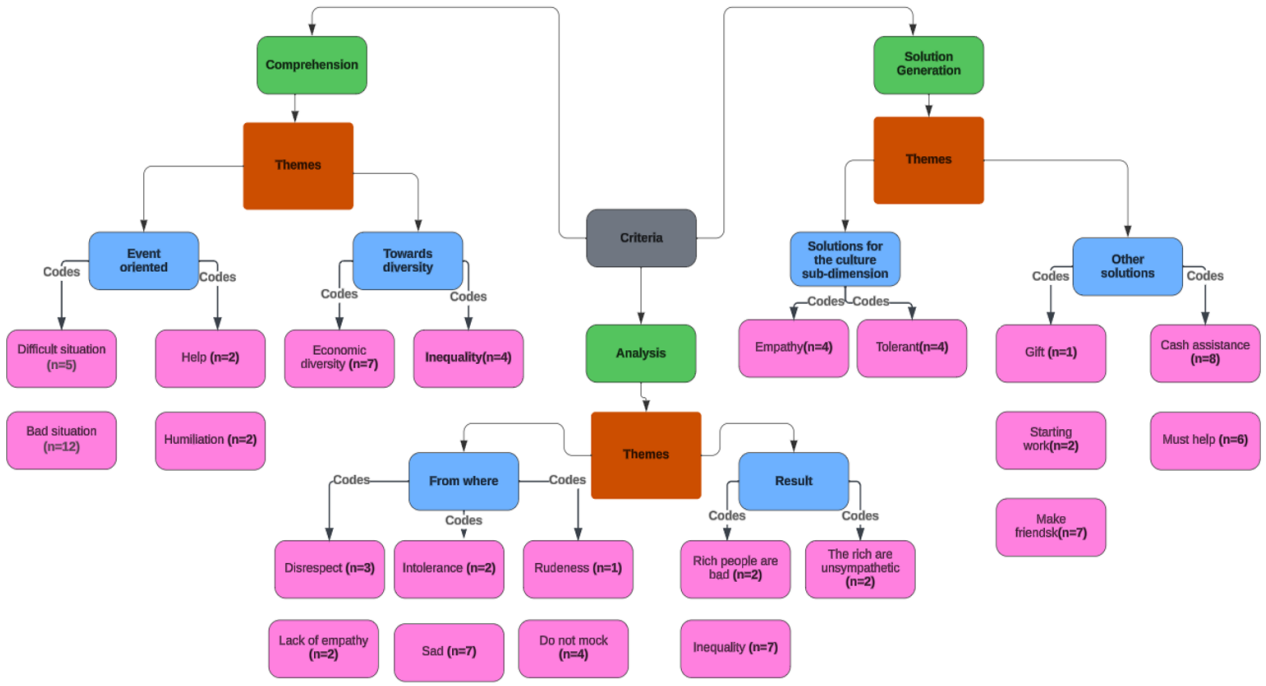

Figure 8. Codes and themes about tolerance awareness towards socio-economic difference scenarios before the action research (Source: Authors' own elaboration)

Socio-economic differences: Figure 8 shows findings about socio-economic differences. In general, students reacted to the situation experienced by the scenario. The responses given at the comprehension level were generally categorized under the event's theme. The students focused more on the cause and did not give many examples of the result. The tolerance-based responses expected from the students were limited at the level of generating solutions. The following quotation is an example of this finding.

> "I do not think they should behave like that; they should empathize with him and treat him well and support him both financially and morally and not make fun of him" (S8).

The most repeated codes in the analysis dimension are "sad, inequality." Examples of student responses in the analysis dimension are, as follows:

> "I did not find this situation pleasant at all because the child who was mocked cannot empathize because he is rich" (S17).

> "They should show a little empathy and respect him and help him" (S21).

Regarding the solution generation, some sample quotations are, as follows.

> "Students at school should be more tolerant, they should not be spoiled" (S16).

The findings show that they generally commented on the badness and difficulty of the event. Interpretations based on socio-economic inequality were very limited. The participant students generally suggested that empathy should be established and help should be provided.

Levels of Students' Mathematical Reasoning Skill

The scores for mathematical reasoning skills before the action are presented in **Appendix A**. Our analysis regarding different representations revealed that two students were at a very low level, six at a low level, one at an intermediate level, twelve at a good level, and the remaining three at a very good level.

The reasoning prediction results revealed that two were at a very low level, seven were at a low level, seven were at an intermediate level, four were at a good level, and the remaining four were at a very good level. The results regarding reasoning deciding demonstrated that one of them was at a very low level, nine were at a medium level, nine were at a good level, and the remaining four were at a very good level.

Our findings regarding solving non-routine problems indicated that ten participants were at a very low level, three of them were at a low level, nine of them were at a medium level, one of them was at a good level, and the remaining one was at a very good level.

The reasoning pattern recognition results revealed that six were at a very low level, ten were at a low level, four were at an intermediate level, and one was at a good level. In addition, the results for the generalization of mathematical reasoning imply that one of them was at a very low level, two of them at a low level, nine of them at a medium level, four of them at a good level, and the remaining eight at a very good level. Furthermore, the findings about mathematical reasoning developing logical ways showed that six were at a very low level, four were at a low level, and twelve were at a medium level. Two of them were at a good level.

After the Research

This section includes findings after the action research.

The development of tolerance

Students' responses about developing awareness of tolerance at the comprehensive level were analyzed after action research (**Appendix A**).

Our findings showed that five students gave acceptable responses in the sub-dimension of culture, eight in the sub-dimensions of race-origin and language, fourteen in the sub-dimension of belief, sixteen in the sub-dimension of gender, and five in the sub-dimension of socio-economic level. The codes and sub-themes at the comprehension level are given in **Appendix B**.

Our findings revealed that the students defined the event in general at the comprehension level before the research; they stated that differences in culture, race origin, gender, belief, language, and socio-economic level caused the problem experienced after action research.

The participants' answers about the development of tolerance awareness at the analysis level are given in **Appendix A**. The results indicated that thirteen students responded in the sub-dimensions of culture and socioeconomic level, and fourteen responded as expected in the sub-dimensions of race-origin, language, belief, and gender. After the action research, we found an increase in almost all students. The codes and themes at the analysis level are given in **Appendix B**.

Cultural differences: The findings about cultural differences were grouped under the themes "towards the event" and "towards difference" (see **Table B1** in **Appendix B**). The most repeated codes at the comprehension level are "cultural difference, lifestyle difference, bad event, the difficult situation." One example quotation is, as follows:

> "It is very difficult for the hero in the scenario to leave his culture aside and adapt to Turkish culture as soon as he arrives" (S14).

The most repeated code at the analysis level of awareness of the development of tolerance for cultural differences is "cultural harmony." This code is followed by the code "tolerance."

The findings reveal three odes to generating solutions. An example quotation at the level of producing solutions is, as follows:

> "His friends should take him with them, empathize with him, and treat him tolerantly because he is from a different culture" (S19).

Race-origin differences: The most frequent code at the comprehension level about racial-origin differences is "racism" (see **Table B1** in **Appendix B**). This code is followed by "injustice, the bad situation." An example is, as follows;

"Discrimination was made because the protagonist in the scenario was black" (S22).

The " equality " code is the most repeated at the analysis level (**Table B2** in **Appendix B**). This is followed by the code "feeling good." Examples of students' statements at the analysis level are, as follows:

"Racial origin difference is not a reason for human discrimination" (S1).

"People of different race-origin should not be discriminated against" (S13).

The most repeated code at the level of producing solutions is "equality." This code is followed by "tolerance" and "racism is bad." Examples of students' statements at the solution generation level;

"We should be kind and generous to people of different racial backgrounds" (S21).

"I think the cult behavior of the police towards people of different racial backgrounds is not right" (S16).

Language differences: The most repeated code at the comprehension level is "bad situation" (**Table B1 in Appendix B**). This code is followed by codes of "language difference" and "helping". A sample quotation for this finding is below:

"Everyone can experience the situation that the hero in the scenario experiences because the language of another country is not learned immediately" (S14).

The most repeated codes at the analysis level are "empathy," "helping," and "not making fun" (**Table B2** in **Appendix B**). Examples from students' answers are, as follows:

"Before making fun of people who speak different languages, they should empathize and understand them and help them instead of making fun of them" (S20).

"It is a very bad situation for me because people who make fun of people because they do not know the language may also fall into such a situation" (S16).

The most recurring codes at the level of generating solutions are "cooperation" and "tolerance" (**Table B3** in **Appendix B**). Examples quotations for this finding are, as follows:

"People who speak different languages should be tolerated and empathized with" (S7).

"They should find solutions by empathizing with people who speak different languages" (S19).

Differences in beliefs: The most repeated code at the comprehension level is "difference of belief." This code is followed by the codes "bad situation," "privilege," and "sadness" (**Table B1** in **Appendix B**). Examples of this finding are, as follows:

"He experienced a bad event and was discriminated against because of his belief" (S8).

"The hero in the scenario experienced a difficult situation because of his belief" (S6).

The most repeated codes at the analysis level are "respect for different beliefs" and "tolerance to different beliefs" (**Table B2** in **Appendix B**). Examples are, as follows:

"I think people with different beliefs are good because differences are beautiful" (S16).

"People with different beliefs and religions should be respected" (S18).

The most repeated codes at the level of generating solutions are "tolerance" and "empathy" (**Table B3** in **Appendix B**). Examples are, as follows:

"They should not approach him/her with prejudice because he/she has different beliefs" (S3).

"Another type of racism is not treating people with different beliefs with tolerance" (S19).

Gender differences: The most recurring code at the comprehension level is "gender discrimination" (**Table B1** in **Appendix B**). This code is followed by "sad situation" and "injustice". Example of students' expressions is, as follows:

"It is a very bad thing that the principal discriminates based on gender; if a boy can do it, so can a girl" (S12).

The most repeated codes at the analysis level s are "there should be no gender discrimination, everyone is equal" (**Table B2** in **Appendix B**). An example at the analysis level is presented below;

"It is ridiculous because there are many people in the world, and not all of them are of the same gender" (S14).

The most repeated codes at the level of generating are "gender discrimination should not be practiced" and "the sad situation should be saved" (**Table B3** in **Appendix B**). The example is, as follows:

> "Zeynep should be selected as the general manager regardless of gender because she is hardworking" (S6).

Socio-economic differences: The most repeated code at the comprehension level is "bad event" (**Table B1** in **Appendix B**). This code is followed by "economic difference" and "not mocking." Examples of students' statements at the comprehension level are, as follows:

> "The friends of the protagonist in the scenario looked at the protagonist with prejudice because he was poor" (S19).

The most repeated codes at the analysis level are "not making fun of," "tolerance," and "they are doing wrong," respectively (**Table B2** in **Appendix B**). Examples of students' statements at the analysis level;

> "It is ridiculous because a person is a human being whether he/she is poor or rich" (S12).

The most repeated codes at the level of generating are "bad event," "should help," and "should be a friend" (**Table B3** in **Appendix B**). The example is, as follows:

> "They should empathize, try to be friends, and act tolerant; they should not make fun of the hero's clothes in the scenario. After all, everyone can be poor one day" (S16).

The results of developing tolerance awareness were obtained from action research at the solution generation level (**Appendix A**). At the end of the action research, seventeen students responded in the culture sub-dimension, sixteen in the race-origin and language sub-dimensions, nineteen in the belief sub-dimension, twenty in the gender sub-dimension, and seventeen in the socio-economic level sub-dimension. Almost all the students showed increased awareness levels towards the development of tolerance. The codes and themes at the solution generation level are given in **Appendix B**.

Regarding generating solutions, students generally suggested solutions unrelated to the sub-dimension of multiculturalism before the research. At the same time, they started to produce solutions related to the sub-dimension after the research.

Development of Mathematical Reasoning Skills

At the end of the action research, the student's scores for mathematical reasoning skills are presented in **Appendix A**. We found that many students (n = 16) were at a very good level regarding reasoning skills. Regarding estimation skills, we found that half of the students developed good reasoning skills. Concerning deciding the correctness of the solution, our findings revealed that five were at a very low level, seven were at a low level, three were at a medium level, and seven were at a good level. Also, four were at a very low level, six at a low level, one at a medium level, five at a good level, and the remaining seven at a very good level.

Regarding recognizing patterns, we found that one was at a low level, four were at a medium level, three were at a good level, and the remaining fifteen were at a very good level. For generalization sub-skills, the results revealed that one of them was at a very low level, one of them was at a low level, three of them were at a medium level, fourteen of them were at a good level, and the remaining four were at a very good level. Regarding developing logical ways for solution, our results indicated that two were at a very low level, one was at a low level, two were at a medium level, seven were at a good level, and the remaining eleven were at a very good level.

After action research, the results showed that more than half of students (n = 17) increased their scores in the sub-dimensions of mathematical reasoning skills in the sub-dimensions of different representations, four in the sub-dimension of deciding the correctness of the solution path, twenty-one in the sub-dimension of recognizing patterns, twenty in the sub-dimension of different representation of the same data, and fifteen in the sub-dimension of estimation. In addition, we found that the total score increased in all sub-dimensions after the action research.

In addition to these findings, the results showed that students participating in this research expressed that using FCM was highly beneficial during the interviews. They found it engaging and enjoyable, improving their productivity in the home study environment. The group work within FCM was particularly well-received, with many students stating that it contributed to a better understanding of the lessons. The following quotes from the interviews support these findings.

> "I realized I was doing mathematics questions without understanding and planning before studying. Through this research, I understand mathematics questions better and can plan solutions to problems" (S1).

> "Before the study, I did not understand and liked the mathematics course. I had prejudices against the course. Thanks to the study, I understand and like mathematics better" (S3).

> "The activities for mathematics reasoning skills helped me, and I learned to interpret the questions. Now, I can solve the questions more easily" (S15).

DISCUSSION

Development of Mathematical Reasoning Skills

Before the action research, we found that students could not solve the problems they encountered using mathematical reasoning skills. Also, our results revealed that the student's mathematical reasoning skill levels were low. Specifically, students' estimation skills were low before the action, and the results after our action research showed an increase. Similarly, students' skill scores for solving non-routine problems were low before the research. After the action research, the results revealed that our research contributed to developing skills in solving non-routine problems.

Regarding the correctness of the solution path, students were insufficient before the research. After research, the results demonstrated that the student's skill scores in deciding the correctness of the solution did not increase. The low level of students' skills in deciding the correctness of the solution path before the action research can be interpreted as the fact that high-level thinking skills are not gained effectively enough. This result supports the findings of Yazgan (2007).

Concerning developing logical solutions, the results showed that the student's skill scores were at a low level before the research. After the action research, we found an increase in the development of logical solutions. While the students could not make any judgment about the questions asked in general before the action research, they could make rational inferences about the problem and develop solutions after the study.

Development of Tolerance

The results showed that students' awareness of tolerance to cultural differences were low in comprehension, analysis, and solution generation sub-dimensions. The findings from interviews revealed that the students generally accepted th problemwith a multicultural school environment and this results meant that there was a need for more awareness about multiculturalism. After the action research, students' awareness of tolerance and cultural differences increased. In addition, we observed that students' awareness to identify the existing problem improved during the process. Similarly, based on rubric analysis and students' responses to the scenarios, we found that their comprehension of the problem also increased. During the interviews, students stated that they realized they had conflicted with different cultures before the action research. In general, they indicated that they tried to behave tolerantly towards people from different cultures after participating in this research.

For example, in results regarding racial-origin differences, we found that the students had low levels of tolerance awareness before the action research. The results showed an increase in students' tolerance awareness. After the action research, we found that students' race-origin awareness levels improved in the comprehension, analysis, and solution-generation stages. Our results support the findings of Brandwein and Donoghue (2011).

Regarding language differences, tolerance awareness was low before this research. Before the research, the students had prejudices against people speaking different languages. However, after the research, they did not have prejudices and see a threat from other people speaking different languages. This situation can be explained by the development of tolerance among the participants. The participants had first heard about Farsi and Arabic languages from their classmates.

Concerning differences in beliefs, our findings showed that before the action research, we found that students were unaware of tolerance. This finding may be because the students generally did not encounter people of different beliefs in their social lives. After conducting the research, the interview results indicated an increase in the student's awareness of differences in beliefs. This result shows a development in their awareness of differences in beliefs. A possible reason for the results may be that the effect of the conservative culture in the region where the students live and the fact that there are only male students in their schools can be considered for the low level of awareness towards gender differences. Furthermore, our results suggested an increase in gender differences. This result shows that our action research developed students' awareness of gender differences. The findings regarding socioeconomic status revealed that students' awareness was low. For this result, a possible explanation may be that the students in this research generally did not encounter people from different socio-economic levels in their schools and classes; they may not have developed an awareness of this issue. This research showed that students' awareness increased after having activities regarding multicultural mathematics education supported by flipped learning.

CONCLUSION

This study has focused on developing students' awareness of tolerance towards cultural differences through flipped learning accompanied by multicultural education using the action research method. This research used the action research method to involve middle school students from a multicultural context. Our results demonstrated that a multicultural mathematics education accompanied by flipped learning helped to improve middle school students' awareness of tolerance concepts. Moreover, using the action research method, our teaching process contributed to developing students' mathematical reasoning skills and views of multicultural education and multiculturalism. We believe our results could be the basis of future research examining the development of flipped-based multicultural mathematics education.

Given that no existing research exists on using the FCM and multicultural education in teaching mathematics, our research contributed to the literature and knowledge of researchers by demonstrating the development of flipped learning in multicultural mathematics education. In addition, flipped learning aligns well with the goals of multicultural mathematics education, which aims to integrate different cultural perspectives into the teaching and learning processes. From this perspective, our findings provide insightful results for researchers and teachers about using flipped learning in a multicultural mathematics education

context. From this point of view, our findings present new information that interests researchers. Our results also show how teaching methods like flipped learning can be changed and adapted in line with the principles of multicultural education. Thus, this present research filled out a gap in literature. In terms of this contribution, this study has uncovered valuable insights into the role of flipped learning and culture in shaping mathematical learning experiences using the flipped learning method. Thus, educators and teachers can create inclusive learning environments that promote engagement, empowerment, and academic success by using flipped learning and recognizing students' diversity and cultural backgrounds. Thus, teachers and educators can strive for more equitable and inclusive mathematics education for all learners.

Recommendations

First, regarding mathematical reasoning skills, students' lack of skills implies a need to emphasize developing their reasoning skills through well-designed interventions. Second, based on our results, we recommend using multicultural and flipped learning activities in multicultural mathematics education. Hence, future studies should be conducted to develop different skills and values, merging flipped learning and multicultural education. Third, in future studies, scholars should conduct similar studies using flipped learning to develop students' understanding of mathematical reasoning skills and other topics in mathematics teaching using multicultural mathematics education. Fourth, further research is needed to examine the long-term development of multicultural education in different contexts using the action research method.

Limitations of the Study

The study was limited to 23 students in the 6th grade at a middle school, and action research process was limited to 10 weeks.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Table A1. Data collection tools

Before implementation	Implementation process	After implementation
Mathematics reasoning level determination form	Camera recordings of the implementation process	Mathematics reasoning skill level determination form
Mathematics reasoning skill level rubric	Student worksheet to be used in practice	Mathematics reasoning skill level rubric
Awareness level determination form for the development of tolerance	Student diaries for each activity	Awareness level determination form for the development of tolerance
Awareness level rubric for tolerance development	Researcher diaries	Awareness level rubric for tolerance development
Semi-structured interview form for pre-implementation	Semi-structured interview about the implementation process	Semi-structured interview form for teaching evaluation
Personal information form	Scenarios	

Table A2. Scoring criteria for mathematical reasoning skills

Points	Levels
0.00-0.99	Very low
1.00-1.99	Low
2.00-2.99	Middle
3.00-3.99	Good
4.00-5.00	Very good

Table A3. Extended scoring criteria for mathematics reasoning skill level

Points	Levels
0.00-2.99	Very low
3.00-5.99	Low
6.00-8.99	Middle
9.00-11.99	Good
12.00-15.00	Very good

Table A4. Descriptive statistics and Kendall tau-b correlations for researchers' analyses (n = 23)

		Average	Standard deviation	Researcher 1	Researcher 2
Before action	Researcher 1	11.78	4.123		.868**
	Researcher 2	11.52	3.716		
Action process	Researcher 1	21.65	5.262		.912**
	Researcher 2	21.35	5.184		
End of action	Researcher 1	26.91	5.239		.868**
	Researcher 2	26.30	4.949		

Note. According to the Kendall tau-b correlation analysis, there were significant positive relationships between the scores of the two researchers who evaluated the scenarios applied before the action, during the action process, and at the end of the action ($\tau b = .868$, $p < .01$; $\tau b = .912$, $p < .01$; and $\tau b = .868$, $p < .01$, respectively)

Table A5. The development of tolerance awareness towards multiculturalism

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism																	
	Culture difference			Race-origin difference			Language differences			Differences in beliefs			Gender difference			S-ED		
	C	A	PS	C	A	PS	C	A	PS	C	A	PS	C	A	PS	C	A	PS
S1	0	0	1	0	0	0	0	1	0	1	0	0	1	0	1	0	1	0
S2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1
S3	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
S5	0	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	1
S6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
S7	0	0	1	0	1	2	1	1	1	0	1	1	1	1	1	0	0	2
S8	0	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1
S9	0	1	1	1	1	1	0	0	1	1	1	1	1	0	1	0	0	1
S10	0	1	1	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0
S11	0	1	0	1	1	0	1	1	1	0	0	0	1	1	1	1	1	1
S12	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
S13	0	0	0	0	1	1	0	1	1	1	0	1	0	1	1	1	0	1

Table A5 (Continued). The development of tolerance awareness towards multiculturalism

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism																	
	Culture difference			Race-origin difference			Language differences			Differences in beliefs			Gender difference			S-ED		
	C	A	PS	C	A	PS	C	A	PS	C	A	PS	C	A	PS	C	A	PS
S14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S15	0	0	1	1	0	1	0	0	1	0	0	1	1	0	0	0	0	1
S16	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1	0	1	2
S17	0	1	1	1	1	1	1	1	1	0	1	1	0	1	1	0	1	1
S18	0	1	1	1	0	1	0	0	0	1	1	1	1	1	1	0	1	1
S19	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
S20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S21	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1
S22	0	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1
S23	0	1	1	1	0	0	0	0	1	0	1	1	0	0	1	0	1	1
S24	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0

Note. S-ED: Socio-economic differences; S: Student; C: Comprehension; A: Analysis; & PS: Produce solutions

Table A6. Students' mathematical reasoning skill levels before the action research

S	Mathematics reasoning subskills							Total points
	Different representations of the same data	Forec-asting	Deciding on the correctness of the solution and the outcome	Solving non-routine problems	Recognizing patterns	Generalization	Develop logical ways to find solutions	
S1	5	2	8	1	3	6	3	28
S2	8	5	0	5	0	0	0	18
S3	12	14	12	7	8	14	0	67
S4	3	3	10	0	2	13	2	33
S5	0	7	11	13	4	9	6	50
S6	5	11	10	10	9	12	7	64
S7	9	6	7	4	4	6	6	42
S8	5	4	12	2	2	9	3	37
S9	12	14	12	7	8	14	0	67
S10	0	4	6	1	2	9	7	29
S11	11	9	11	7	8	6	9	61
S12	11	5	11	8	5	3	5	48
S13	10	9	10	2	3	13	8	55
S14	9	1	9	2	3	8	3	35
S15	9	7	11	7	5	9	6	52
S16	12	13	12	1	3	9	8	58
S17	11	10	6	1	6	6	6	46
S18	9	6	7	4	4	6	6	42
S19	10	8	7	8	0	7	7	47
S20	10	5	6	8	5	15	6	55
S21	10	8	7	8	0	7	7	47
S22	5	7	11	2	3	7	2	37
S23	9	3	6	1	4	5	1	29
S24	5	12	13	8	7	15	10	70

Note. S: Student

Table A7. Students' scores about the development of awareness of tolerance in comprehension level

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism					
	Culture difference	Race-origin difference	Language differences	Differences in beliefs	Gender difference	S-ED
S1	1	1	1	2	2	1
S2	0	1	1	0	0	0
S3	0	2	2	1	1	0
S4	0	0	0	0	1	0
S5	2	1	1	2	2	2
S6	2	1	1	1	2	1
S7	2	1	1	1	1	1
S8	1	1	1	2	2	1
S9	1	1	1	2	2	1

Table A7 (Continued). Students' scores about the development of awareness of tolerance in comprehension level

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism					
	Culture difference	Race-origin difference	Language differences	Differences in beliefs	Gender difference	S-ED
S10	0	1	1	2	0	1
S11	1	1	1	1	2	1
S12	1	2	2	2	2	2
S13	1	1	1	2	2	1
S14	2	2	2	2	2	2
S15	1	2	2	1	2	1
S16	1	2	2	2	2	1
S17	2	1	1	2	2	0
S18	1	2	2	2	2	1
S19	0	0	0	0	0	1
S20	1	2	2	2	2	2
S21	0	1	1	1	2	1
S22	1	2	2	2	2	2
S23	1	1	1	2	0	1
S24	-	-	-	-	-	-

Note. S: Student & S-ED: Socio-economic differences

Table A8. Students' scores about tolerance awareness in the analysis level

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism					
	Culture difference	Race-origin difference	Language differences	Differences in beliefs	Gender difference	S-ED
S1	2	2	2	2	2	2
S2	0	0	0	0	0	0
S3	1	1	1	1	1	1
S4	0	0	0	0	0	0
S5	2	2	2	2	2	2
S6	2	2	2	2	2	2
S7	1	1	1	1	1	1
S8	1	1	1	1	1	1
S9	1	1	1	1	1	1
S10	1	1	1	1	1	1
S11	2	2	2	2	2	2
S12	1	1	1	1	1	1
S13	2	2	2	2	2	2
S14	2	2	2	2	2	2
S15	2	2	2	2	2	2
S16	2	2	2	2	2	2
S17	2	2	2	2	2	2
S18	1	1	1	1	1	1
S19	2	2	2	2	2	2
S20	2	2	2	2	2	2
S21	1	1	1	1	1	1
S22	2	2	2	2	2	2
S23	2	2	2	2	2	2
S24	-	-	-	-	-	-

Note. S: Student & S-ED: Socio-economic differences

Table A9. Students' scores about tolerance awareness towards multiculturalism in the level of solution generation

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism					
	Culture difference	Race-origin difference	Language differences	Differences in beliefs	Gender difference	S-ED
S1	1	1	1	1	1	1
S2	0	0	0	0	0	0
S3	2	2	2	2	2	2
S4	1	1	1	1	1	1
S5	2	2	2	2	2	2
S6	2	2	2	2	2	2
S7	1	1	1	1	1	1

Table A9 (Continued). Students' scores about the development of awareness of tolerance in comprehension level

S	Awareness levels of the development of tolerance for the sub-dimensions of multiculturalism					
	Culture difference	Race-origin difference	Language differences	Differences in beliefs	Gender difference	S-ED
S8	2	2	2	2	2	2
S9	2	2	2	2	2	2
S10	2	2	2	2	2	2
S11	2	2	2	2	2	2
S12	2	2	2	2	2	2
S13	2	2	2	2	2	2
S14	2	2	2	2	2	2
S15	2	2	2	2	2	2
S16	2	2	2	2	2	2
S17	2	2	2	2	2	2
S18	2	2	2	2	2	2
S19	2	2	2	2	2	2
S20	2	2	2	2	2	2
S21	2	2	2	2	2	2
S22	0	0	0	0	0	0
S23	1	1	1	1	1	1
S24	-	-	-	-	-	-

Note. S: Student & S-ED: Socio-economic differences

Table A10. Students' mathematical reasoning skill scores after action research

S	Mathematics reasoning subskills					
	Different representations of the same data	Forecasting	Deciding on the correctness of the solution and the outcome	Solution of non-routine problems	Recognizing patterns	Generalization
S1	14	6	4	10	7	7
S2	11	5	5	8	5	3
S3	10	5	0	0	7	9
S4	4	5	0	2	12	0
S5	15	12	10	14	14	9
S6	15	6	6	10	9	9
S7	10	4	1	4	12	9
S8	15	11	4	9	11	9
S9	15	13	11	14	15	15
S10	8	8	4	1	12	6
S11	15	14	7	15	15	12
S12	15	12	11	13	12	10
S13	14	3	6	5	12	9
S14	15	11	4	5	8	11
S15	13	13	4	14	11	15
S16	15	15	9	14	15	10
S17	10	13	9	14	15	9
S18	15	11	6	5	12	9
S19	13	8	9	10	10	9
S20	15	15	11	11	15	12
S21	2	6	2	4	7	6
S22	10	9	0	1	15	9
S23	13	9	5	4	14	9
S24	-	-	-	-	-	-

Note. S: Student

APPENDIX B

Table B1. The sub-themes and codes obtained at the level of comprehension of students' awareness of tolerance towards multiculturalism

Criteria	Themes	Codes	f
Culture difference tolerance development awareness comprehension	Case study	Bad event	3
		Difficult situation	3
		Discrimination	2
		Gets used to it in time	2
	Towards diversity	Culture difference	7
		Lifestyle differences	4
Race-origin difference tolerance development awareness comprehension	Case study	What the police are doing is wrong	2
		Discrimination	2
		Bad situation	4
		Injustice	4
	Towards diversity	Racism	16
Language difference tolerance development awareness comprehension	Case study	Making fun of	6
		Helping	8
		Empathize	5
		Bad situation	11
		Tolerant behavior	3
	Towards diversity	Language differences	9
Differences of belief tolerance development awareness comprehension	Case study	Bad situation	4
		Not making fun of	1
		Privilege	2
		Injustice	1
		Do not worry	2
		It is a difficult situation	1
	Towards diversity	Differences in beliefs	12
Gender difference tolerance development awareness comprehension	Case study	Sad situation	1
		Injustice	1
	Towards diversity	Gender discrimination	21
Socio-economic difference tolerance development awareness comprehension	Case study	Not mocking	5
		Empathize	4
		Tolerance	2
		Misbehavior	2
		Bad event	13
	Towards diversity	Economic diversity	8
		Rich part	2

Table B2. The codes and themes on the development of awareness of tolerance towards multicultural education in the level of analysis

Criteria	Themes	Codes	f
Culture difference tolerance development awareness analysis	From where	Culture alignment	7
		Empathy	3
		Tolerance	5
		Equality	4
		Respect	2
	Conclusion	Gets used to it in time	3
		Must help	4
		Must not mock	4
Race-origin difference tolerance development awareness analysis	From where	To black people we are all different	1
		Positive statements	2
	Conclusion	Help	3
		Tolerance	3
		Feeling good	5
		Equality	15
Language difference tolerance development awareness analysis	From where	No need to know the language	1
		Something wrong	4
		Something bad	4
	Conclusion	Empathize	9
		Behaving in a tolerant manner	4
		Helping	7
		Must not mock	7
		Google translation assistant	4

Table B2 (Continued). The codes and themes on the development of awareness of tolerance towards multicultural education in the level of analysis

Criteria	Themes	Codes	f
Differences of belief tolerance development awareness analysis	From where	Forcing Islam	1
		Forcing Islam Invitation to Islam	3
	Conclusion	Equality	7
		Respect for different beliefs	12
		Tolerance for different faiths	4
Gender difference tolerance development awareness analysis	From where	Girls and boys are not equal	2
		Sad	3
	Conclusion	There should be no gender discrimination	13
		Everyone is equal	8
		Must be tolerant	1
Socio-economic difference tolerance development awareness analysis	From where	Not mocking	8
		They are doing it wrong	5
		People are equal	4
	Conclusion	Being tolerant	5
		Empathizing	3
		Helping the poor	3

Table B3. The themes and codes of the development of tolerance awareness towards multiculturalism at the level of solution generation

Criteria	Themes	Codes	f
Culture difference tolerance development awareness to the problem generating solutions	Solutions for the culture sub-dimension	Teach culture	5
		Tolerance	12
		Helping	6
		Empathy	8
	Other solutions	Gets used to it in time	3
Race-origin difference tolerance development awareness to the problem generating solutions	Race-origin practices for the sub-dimension	Racism is bad	4
		Empathy	3
		Tolerance	4
		Equality	15
		Discrimination	3
	Other solutions	To make happy	2
Differences of belief tolerance development awareness to the problem generating solutions	Solutions for the belief sub-dimension	Tolerance	11
			9
			1
			3
			2
	Other solutions	Empathy	5
Gender differences tolerance development awareness to the problem generating solutions	Solutions for the belief sub-dimension	There should be no gender discrimination	15
	Other solutions	Save from the sad state of affairs	4
		No injustice should be done	3
Socio-economic difference tolerance development awareness to the problem generating solutions	Socio-economic practices for sub-dimension	Empathy	6
		Tolerance	6
		Must help	7
		Must be a friend	7
		Equal	3
		No mockery	5
		Empathize	5
		Tolerance	4
	Other solutions	Not mocking	2
		Misbehavior	2
		Bad event	13
		Focus on the good	2
Language differences tolerance development awareness to the problem generating solutions	Practices for language sub-dimension	Tolerance	10
		Empathy	8
		Assistance	14
		Learning a foreign language	4
		No mockery	5
	Other solutions	Communication through body language	4