International Electronic Journal of Mathematics Education
2025, 20(1), em0805
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Construction and analysis of test in triangle and circle trigonometry

Dina Kamber Hamzić [1]*, Mirsad Trumić [2], Ismar Hadžalić [1]

[1] Faculty of Science, University of Sarajevo, Sarajevo, BOSNIA AND HERZEGOVINA
[2] Agricultural and Medical School, Brčko District, BOSNIA AND HERZEGOVINA
***Corresponding Author:** dinakamber@pmf.unsa.ba

Citation: Kamber Hamzić, D., Trumić, M., & Hadžalić, I. (2025). Construction and analysis of test in triangle and circle trigonometry. *International Electronic Journal of Mathematics Education, 20*(1), em0805. https://doi.org/10.29333/iejme/15734

ARTICLE INFO

Received: 22 May 2024

Accepted: 29 Nov. 2024

ABSTRACT

Trigonometry is an important part of secondary school mathematics, but it is usually challenging for students to understand and learn. Since trigonometry is learned and used at a university level in many fields, like physics or geodesy, it is important to have an insight into students' trigonometry knowledge before the beginning of the university courses. This research aimed to develop a test in triangle and circle trigonometry, which can be used to test students' prior knowledge of basic trigonometric concepts. A test with multiple-choice questions was developed based on content and learning outcomes from the secondary school curriculum. After a group of three experts evaluated the content validity of the test, the corrected version was given to 58 first-year students at the Department of Mathematics and Computer Science. After the test and item analysis, a final version of the test was made which can be used as a short initial test to measure students' prior knowledge of trigonometry at the beginning of a university course.

Keywords: circle trigonometry, reliability, triangle trigonometry, validity

INTRODUCTION

Trigonometry is an important area of mathematics. It is used in astronomy, physics, architecture and construction, geodesy, forensics, acoustics, etc. Trigonometry is an indispensable part of secondary school and university mathematics, and as a field that connects geometry, algebra, and graphics, it is an important introduction to differential and integral calculus (Weber, 2008).

Usually, students first learn about trigonometric functions as ratios of side lengths in a right-angled triangle. This approach is good in geometry problems where unknown sides or angles are searched for, but if one needs to determine the value of a trigonometric function for some argument, determine whether it increases or decreases at some interval, draw a graph of the function–this approach does not help (Weber, 2008). To answer these questions, students have to understand trigonometric functions as functions, not as side length ratios. To define trigonometric functions for arbitrary real numbers, we define trigonometric functions using a unit circle.

Chin and Tall (2012) introduce three contexts of understanding trigonometry: *triangle trigonometry* (includes problems connected with the relations of sides and angles in a right-angled triangle), *circle trigonometry* (includes angles of any size with a vertex at the center of the circle, and properties of trigonometric functions), and *analytic trigonometry* (includes trigonometric functions expressed with Taylor series and the use of complex numbers to connect exponential and trigonometric functions).

In the Bosnia and Herzegovina educational system, the study of trigonometry begins in secondary school. Regardless of the school choice (vocational, technical, or grammar school), all students encounter triangle trigonometry and the basics of circle trigonometry. In grammar schools and some technical schools, students also learn addition formulas and formulas derived from them, they learn how to solve trigonometric equations and inequalities, and how to draw graphs of trigonometric functions. Analytic trigonometry is learned only in mathematical and engineering studies at the university level.

The goal of this paper is to develop a test for checking student achievements in triangle trigonometry and the basics of circle trigonometry–those parts of trigonometry that all students should know at the end of their secondary education. Test results will also provide insight into students' basic trigonometry knowledge before starting their university courses. Below will be presented in more detail the topics covered by the test, appropriate learning outcomes and standards, basic test theory, and the process of constructing the test itself.

This study was conducted as a part of Ismar Hadžalić's master thesis titled "Konstrukcija validnog i pouzdanog testa iz trigonometrije trougla i kružnice" [Construction of a valid and reliable test in triangle and circle trigonometry].

Copyright © 2025 by Author/s and Licensed by Modestum. This is an open access article distributed under the Creative Commons Attribution License which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Table 1. Trigonometry standards, with corresponding level, area, and component

Standard	Level	Area and component
The student connects basic trigonometric relations with the sketch and vice versa (calculates the values of the trigonometric functions).	Basic	Area: Algebra & Component: Algebraic expressions, functions, proportions, and their applications
The student converts degrees to radians, and vice versa.	Basic	Area: Geometry and measurements & Component: Measures and measurements
The student applies triangle trigonometry in simple situations.	Intermediate	Area: Geometry and measurements & Component: Measures and measurements
The student connects and applies knowledge of trigonometry.	Advanced	Area: Algebra & Component: Algebraic expressions, functions, proportions, and their applications
The student converts mathematical models into new ones to solve a given problem.	Advanced	Area: Algebra & Component: Algebraic expressions, functions, proportions, and their applications
The student solves geometric problems using trigonometry.	Advanced	Area: Geometry and measurements & Component: Shapes, solids, and transformations

Triangle Trigonometry and Circle Trigonometry

The recommended content of the basics of trigonometry includes the following (Ministry of Education of Sarajevo Canton, 2022): definitions of trigonometric functions in a right-angled triangle; oriented angles; radian; unit circle; definitions of trigonometric functions in a unit circle; values of trigonometric functions of angles: 0°, 30°, 45°, 60°, 90°, 180°, 270°, and 360°; values of trigonometric functions of arbitrary angles (use of calculators or tables); basic trigonometric identities; periodicity of trigonometric functions; trigonometric functions of a negative argument; even and odd trigonometric functions; reducing trigonometric functions to the first quadrant. The same document lists the following learning outcomes (Ministry of Education of Sarajevo Canton, 2022):

- the student defines trigonometric functions in a right-angled triangle,
- the student explains the relationship between trigonometric functions,
- the student applies formulas in trigonometry, and
- the student analyzes the properties of basic trigonometric functions in a unit circle.

In 2021, the Agency for Pre-Primary, Primary and Secondary Education (APOSO, 2021) published the standards of learning achievements in mathematics, for the end of the third, sixth, and ninth grade of primary school, and the end of secondary education. In standards, mathematics is divided into four areas, each area is divided into two or three components, and every component contains two, three, or four basic learning outcomes. Each outcome is associated with indicators, and the standards of learning achievements in mathematics (APOSO, 2021) joins the standards to these indicators in three levels: basic, intermediate, and advanced standards.

Table 1 lists standards related to the trigonometry content studied in all secondary schools, their level, area, and component to which they belong. Standards relating to the trigonometry content that is not learned in all secondary schools (and will not be covered by this test) are omitted.

About Testing

Tests are used in education to analyze and determine the extent to which students have been able to understand and acquire knowledge from the content being learned at a given time. A useful and significant evaluation of students' achievements is possible only if the used test serves its purpose. When constructing a test, the first step is to determine what we want to measure (DeVellis & Thorpe, 2022). In the planning phase of the test, the objectives of the test must be carefully defined. Also, at this stage, the type of tasks, the scope of the test, the duration of the test, and the method of scoring should be known. To use the test results, they must be valid and reliable, and test items should be sensitive and of appropriate difficulty (Simanjuntak et al., 2019).

Item difficulty and discriminative validity

Item difficulty (P) is the proportion of the subjects who correctly answered that item (Crocker & Algina, 2008) and it can have values between 0 and 1 (including 0 and 1). The closer the P is to 1, the easier the item is (Tandas, 2021). Optimal P is usually taken to be 0.5, but in multiple choice questions, because of the possibility of guessing the answer, the optimal difficulty is higher than 0.5. For example, Crocker and Algina (2008) cite 0.62 as an expected P of a multiple-choice question with four options. There is no single agreement on what an acceptable P range is: according to Kline (2015), an acceptable range is from 0.2 to 0.8, while according to Husremović (2016), any item with difficulty between 0.15 and 0.9 is acceptable.

Item discriminative validity shows how the item differentiates the subjects in relation to what is measured by the test. Item has good disciminative validity if better subjects are more successful than weaker subjects in answering that item (Husremović, 2016). One way of determining the item discriminative validity is using the method of extreme groups, and another way is using item-total correlations (Husremović, 2016).

In the extreme group method, the subjects are divided into groups according to the total results achieved on the test. Groups of (approximately) a third of the best and a third of the weakest subjects are formed. The index of discrimination (D) is calculated by subtracting the proportion of the lower group who answered the item correctly from the proportion of the upper group who answered the item correctly (Crocker & Algina, 2008). This index can only be applied to dichotomously scored items (Crocker & Algina, 2008)–usually, items scored with 0 or 1 (incorrectly/correctly solved item). According to Crocker and Algina (2008) and

Husremović (2016): if the D is lower than 0.2, that item has to be removed from the test; if the index has a value between 0.2 and 0.3, the item should be revised; if the index value is greater than or equal to 0.3, the item is good; if the value is greater than or equal to 0.4, item is very good.

Another way to determine discriminative validity is through the correlation between the item results and the total test results (item-total correlation). If items are dichotomous (only correct and incorrect answers are possible), a point-biserial correlation coefficient (r_{pbis}) is used (Kline, 2015). It is calculated by the following formula:

$$r_{pbis} = \frac{M_p - M_t}{\sigma_t}\sqrt{\frac{P}{Q}}, \tag{1}$$

where M_p is the average test score of subjects who answered this item correctly, M_t is the average test score of all subjects, σ_t is the test standard deviation, and $Q = 1 - P$ (Crocker & Algina, 2008; Husremović, 2016).

If the test has fewer items (for Kline (2015) that is every test with less than 100 items, while for Crocker and Algina (2008) that is a test with less than 25 items), the corrected formula for r_{pbis} has to be used:

$$r_{it}(corrected) = \frac{r_{it}\sigma_t - \sigma_i}{\sqrt{\sigma_t^2 + \sigma_i^2 - 2\sigma_t\sigma_i r_{it}}}, \tag{2}$$

where r_{it} is the item-total correlation, σ_t is the test standard deviation, and σ_i the item standard deviation (Kline, 2015). According to Kline (2015), any item with a value of r_{pbis} greater than 0.2 is acceptable.

Validity and reliability

Validity refers to the degree to which a test measures what it is intended to measure and is one of the basic criteria that a test must meet (Darr, 2005b). There are three main types of validity: content, criterion, and construct validity (Crocker & Algina, 2008).

Content validity refers to the compatibility of the content of the test and the content that should be in the test, i.e., how well the test items represent the domain/content of interest (Crocker & Algina, 2008). Content validity is meaningful only in tests in which the content in question is clear, like in mathematics (Kline, 2015). Content validity is usually assessed by a panel of independent experts who assess whether the test items adequately measure the domain of interest (Crocker & Algina, 2008). Husremović (2016) calls this a qualitative analysis of content validity and mentions a quantitative analysis of content validity through the analysis of test items from the aspect of difficulty and discriminative validity.

Criterion validity refers to the correlation of test results with the results of other tests that are assumed to be theoretically related (Husremović, 2016). There are two types of criterion validity: concurrent and predictive validity (Crocker & Algina, 2008). Concurrent validity is assessed by correlating the test results with the results of another, similar, but valid test, which immediately raises the question–is there a valid test similar to the one being constructed (Kline, 2015)? Predictive validity is assessed by correlating test results with the results of a related test made later in the future. Many psychometrists consider predictive validity to be the most convincing evidence of test efficiency (Kline, 2015).

Construct validity refers to whether a test measures what it needs to measure and not something else that is not the purpose of the test (Darr, 2005b).

Test reliability refers to the consistency of test results in the sense that the results stay similar if the same subjects are tested under the same (or similar) conditions (Crocker & Algina, 2008). Reliability can be observed from three aspects: the aspect of time, the aspect of the marker, and the aspect of test items (Darr, 2005a). The first aspect means that the same or equivalent test is given to the same subjects at two different times, and then the correlation coefficient between the results of those two tests is determined (Darr, 2005a). In an educational setting, this method of reliability verification, also called the test-retest method, is difficult to implement. When we want to test the reliability of test results from the aspect of markers, we compare the test results from several different markers. This is called inter-rater reliability.

In an educational context, however, reliability is usually assessed by methods of internal consistency (Crocker & Algina, 2008). The test reliability is determined based on the results of the test that the subjects did only once, in a way to determine the interconnection between the test items themselves. If the items are interconnected, the test is internally consistent (DeVellis & Thorpe, 2022). The usual measure of test internal consistency used in research is Cronbach's alpha coefficient (Husremović, 2016). Its values go between 0 (inconsistency) and 1 (perfect consistency). DeVellis and Thorpe (2022) give the following scale for the values of Cronbach's alpha coefficient and appropriate internal consistency: if alpha is less than 0.60, it is unacceptable, alpha between 0.60 and 0.65 is undesirable, from 0.65 to 0.70 is barely acceptable, between 0.70 and 0.80 is decent, between 0.80 and 0.90 is very good, and for alpha greater than 0.90 test needs to be shortened, i.e., number of test items must be reduced.

Newer studies (for example, Hayes & Coutts, 2020 or McNeish, 2018) recommend that other consistency measures should be used instead of Cronbach's alpha coefficient, such as McDondald's omega coefficient. McNeish (2018) believes that Cronbach's alpha coefficient is often not the optimal method for assessing reliability, because the assumptions necessary to use this coefficient are too strict and often not met.

Table 2. Item difficulties, indices of discrimination, and point-biserial correlation coefficients

Item number	Item difficulty (P)	Index of discrimination (D)	r_{pbis}
1	0.88	0.28	0.21
2	0.29	0.39	0.32
3	0.64	0.67	0.47
4	0.84	0.28	0.24
5	0.66	0.78	0.59
6	0.60	0.57	0.39
7	0.74	0.61	0.46
8	0.66	0.46	0.35
9	0.67	0.67	0.56
10	0.43	0.69	0.44
11	0.59	0.37	0.22
12	0.45	0.74	0.55
13	0.66	0.67	0.48
14	0.24	0.49	0.44
15	0.24	0.03	-0.09

MATERIALS AND METHODS

Test

The first version of the test had 15 multiple-choice questions, each with four possible answers (only one of which is correct). Three university lecturers reviewed the test for qualitative content validation. Two of them teach pre-calculus courses, in which trigonometry is taught, while the third lecturer teaches trigonometry within the course methods of teaching mathematics. These experts suggested changes in five items that would make them clearer, and replacement of four items, so that the test would be in accordance with the content and standards listed in the section triangle trigonometry and circle trigonometry.

After corrections as suggested by experts, the new test contained 15 multiple-choice questions, some of them new or reworded. Every item was scored with 1 or 0 points, depending on whether it was answered correctly or not. Test items and standards for each item are given in **Appendix A**.

Participants

The research participants were 58 students enrolled in the first year at the department of mathematics and computer sciences. The goal of this research was to check whether the test gives reliable and valid results for the population of first-year students, and what kind of prior knowledge they have from the basics of trigonometry. Since they have just enrolled in the first year, we believe that the test shows their knowledge of trigonometry at the end of secondary education.

All participants were of legal age at the time of the testing, so only their consent was needed, and not from their parents.

Participants were given 45 minutes to do the test, the results of which are given below.

RESULTS AND DISCUSSION

Average Result and Standard Deviation

The maximum possible score on the test was 15, and it was achieved by one student. No student had 0 points as a total score, but one student scored only 1 point. The average test result was 8.59, while the standard deviation was 3.38.

Reliability

McDonald’s omega coefficient was 0.812, which is a good value of internal consistency (Hadianfard et al., 2021).

Predictive Validity

For the 58 participants of this research, the results of this test were compared with the results of a regular trigonometry test done later in the pre-calculus course. The later test evaluated the students’ knowledge of trigonometry taught in the pre-calculus course (trigonometric formulas, inverse trigonometric functions, trigonometric equations, and trigonometric inequalities), and we wanted to see if the results of our test could predict the results in the pre-calculus test. Pearson’s correlation coefficient was $r = .589$, with $p < 0.01$, which means there was a statistically significant and strong correlation between the results of this test and the later pre-calculus trigonometry test.

Item Analysis and Discussion

Values of P, D, and r_{pbis} for all 15 test items are given in **Table 2**. Items with item difficulties outside of the 0.2-0.8 range (Kline, 2015) will be excluded from the final version of the test, as well as the items with a D less than 0.3 (Crocker & Algina, 2008; Husremović, 2016) or with r_{pbis} less or equal to 0.2 (Kline, 2015).

From **Table 2** one can see that items 1, 4, and 15 did not meet the criteria of appropriate difficulty and/or discriminative validity. Item 1 was too easy (P = 0.88), and, as expected, it also had a weak discriminative validity (D = 0.28). This means that it was solved equally well by good and weaker students. A similar situation happened with item 4, which had P = 0.84 and D = 0.28. A possible reason for this was that in secondary school, the majority of students learned by heart that $60°$ is $\frac{\pi}{3}$ in radians (item 1), and that $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$ (item 4).

On the other hand, item 15 was more difficult, but had a surprisingly low D = 0.03, and even had a negative r_{pbis} = -0.09. This meant that this item did not distinguish well between better and weaker students. A more detailed look at the results of this item revealed that of the 29 students who had the best results, only seven answered this item correctly. Of the other 29 students (weaker ones), seven also answered the item correctly.

Of the remaining 12 items, 10 of them had a D greater than 0.4. Item 2 is an exception, it has an index D = 0.39, and difficulty P = 0.29. This item required an understanding of the concept of radian. Several studies on trigonometry learning showed that students had problems understanding radians (e.g., Akkoc, 2008; Moore et al., 2015; Topçu et al., 2006; Tuna, 2013), and the results of this test showed that as well. Nevertheless, item 2 had an acceptable level of difficulty and discriminative validity.

Item 11 also had a D slightly lower than 0.4 (D = 0.37), while its difficulty value (P = 0.59) was very close to the expected value of 0.62 (Crocker & Algina, 2008). Like item 2, it had an acceptable level of difficulty and discriminative validity.

Other items had a D over 0.4, making them very good in their ability to distinguish good from weaker students. Of these ten items, six had difficulty values very close to the expected value of 0.62 (items 3, 5, 6, 8, 9, and 13). They required basic knowledge of trigonometry and knowledge of the properties of trigonometric functions, like even/odd functions, the definition of trigonometric functions in right-angled triangles, and values of trigonometric functions for some specific angle values. More complex calculation was not needed for these items.

Item 7 was a little easier (P = 0.74) and required the knowledge of signs of trigonometric functions. A possible reason why students found this item so easy was the visual representation of the sign of trigonometric functions depending on the quadrant, which is commonly used in our educational system.

Item 14 was difficult (P = 0.24), which was expected because it required more work and calculations than other items. Items 10 and 12 were also somewhat difficult, with difficulty indices P = 0.43 and P = 0. 45, respectively. Both items required the same: recognizing that for different angle values, trigonometric functions can be equal.

Final Test

By eliminating items 1, 4, and 15, the test with 12 items remains, with each item having an acceptable level of difficulty and discriminative validity (most have a very good level of discriminative validity and an expected level of difficulty). Even items that do not on have an "ideal" difficulty or discrimination index, have their place in the test, because with them the essential parts of the content knowledge are checked, and also some characteristics and knowledge of students come to the fore. If only items with a difficulty index of around 0.62 were left in the test, some students would not answer any item correctly, misleading the marker to believe that these students do not know trigonometry at all (Husremović, 2016). Also, the best students could not be determined, because they would have the same score on the test as average students. For this test, with three items removed, the average student score is 6.62 and the standard deviation is 3.19. The value of McDonald's omega coefficient is now 0.792, which is slightly lower than the value of this coefficient for the entire test, but it is still an acceptable value (Hadianfard et al., 2021).

CONCLUSION

This research focused on developing a test in triangle and circle trigonometry - a trigonometry content learned in the first two years of secondary school in Bosnia and Herzegovina. Trigonometry is a very demanding area to teach, and for students, it is usually difficult to understand. The goal was to develop a test that would effectively measure students' understanding of this area, as well as identify possible difficulties or deficiencies in their knowledge.

The first step was to determine the learning outcomes and standards determined by official documents, like the curriculum. Then a preliminary version of the test was made, which was given to a group of three experts who evaluated the content validity of the test. Their remarks and suggestions were implemented in the corrected version of the test.

The corrected version of the test was given to first-year university students. The analysis of items' difficulty indices, indices of discrimination, and point-biserial correlation coefficients led to the final version of this test. The final version of the test can be used to measure basic triangle and circle trigonometry knowledge, because it has a suitable range of items from easier to more difficult ones, all items have good or very good levels of discrimination index, and the obtained results were reliable.

We suggest using this test as a short initial test for measuring students' prior trigonometry knowledge at the beginning of some course, rather than as a final test at the end of a course. Also, it is necessary to keep in mind that reliability and validity are characteristics of the test results, not necessarily the test itself (Fan, 2013). So, before its use in the classroom, one should check the learning outcomes and standards they intend to measure, and also check the results for reliability (for example, calculate McDonald's omega coefficient of the test results).

Author contributions: DKH: conceptualization, formal analysis, investigation, writing — original draft, review and editing; **MT**: writing — original draft preparation; **IH**: conceptualization, methodology, data curation, formal analysis. All authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that the study was approved by the Council of the Faculty of Science–University of Sarajevo on 23 September 2022 with approval number 01/06-2330/2-2022. All participants were adults and informed about the research, its purpose, and anonymity.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Akkoc, H. (2008). Pre-service mathematics teachers' concept images of radian. *International Journal of Mathematical Education in Science and Technology, 39*(7), 857-878. https://doi.org/10.1080/00207390802054458

APOSO. (2021). Standardi učeničkih postignuća za matematiku [Student achievement standards for mathematics]. *Agency for Pre-Primary, Primary and Secondary Education*. https://aposo.gov.ba/sadrzaj/uploads/Matematika-BOS-s-napomenom.pdf

Chin, K. E., & Tall, D. (2012). Making sense of mathematics through perception, operation and reason: The case of trigonometric functions. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 264). Taipei, Taiwan.

Crocker, L., & Algina, J. (2008). *Introduction to classical and modern test theory*. Cengage Learning.

Darr, C. (2005a). A hitchhiker's guide to reliability. *SET: Research Information for Teachers, 3*(5), 59-60. https://doi.org/10.18296/set.0623

Darr, C. (2005b). A hitchhiker's guide to validity. *SET: Research Information for Teachers, 2*, 55-56. https://doi.org/10.18296/set.0639

DeVellis, R. F., & Thorpe, C. T. (2022). *Scale development: Theory and applications* (5th ed.). SAGE.

Fan, X. (2013). "The test is reliable"; "the test is valid": Language use, unconscious assumptions, and education research practice. *The Asia-Pacific Education Researcher, 22*(2), 217-218. https://doi.org/10.1007/s40299-012-0036-y

Hadianfard, H., Kiani, B., Azizzadeh Herozi, M., Mohajelin, F., & Mitchell, J. T. (2021). Health-related quality of life in Iranian adolescents: A psychometric evaluation of the self-report form of the PedsQL 4.0 and an investigation of gender and age differences. *Health and Quality of Life Outcomes, 19*, Article 108. https://doi.org/10.1186/s12955-021-01742-8

Hayes, A. F., & Coutts, J. J. (2020). Use omega rather than Cronbach's alpha for estimating reliability. But … *Communication Methods and Measures, 14*(1), 1-24. https://doi.org/10.1080/19312458.2020.1718629

Husremović, D. (2016). *Osnove psihometrije za studente psihologije* [Basics of psychometrics for psychology students]. University of Sarajevo–Faculty of Philosophy.

Kline, P. (2015). *A handbook of test construction: Introduction to psychometric design* (1st ed.). Routledge.

McNeish, D. (2018). Thanks coefficient alpha, we'll take it from here. *Psychological Methods, 23*(3), 412-433. https://doi.org/10.1037/met0000144

Ministry of Education of Sarajevo Canton. (2022). Kurikulum matematika [Mathematics curriculum]. *Ministry of Education of Sarajevo*. https://kurikulum.ks.gov.ba/sites/default/files/2021-04/Kurikulum_matematika.pdf

Moore, K. C., LaForest, K. R., & Kim, H. J. (2015). Putting the unit in pre-service secondary teachers' unit circle. *Educational Studies in Mathematics, 92*(2), 221-241. https://doi.org/10.1007/s10649-015-9671-6

Simanjuntak, E., Hutabarat, H. D. M., & Hia, Y. (2019). The effectiveness of test instrument to improve mathematical reasoning ability of mathematics student. *Journal of Physics: Conference Series, 1188*, Article 012048. https://doi.org/10.1088/1742-6596/1188/1/012048

Tandas, J. B. (2021). Development of a valid and reliable summative test in plane trigonometry. *Journal of Science and Mathematics Letters, 9*(1), 46-59. https://doi.org/10.37134/jsml.vol9.1.5.2021

Topçu, T., Kertil, M., Akkoç, H., Yilmaz, K., & Önder, O. (2006). Pre-service and in-service mathematics teachers' concept images of radian. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 5* (pp. 281-288). Prague, Czech Republic.

Tuna, A. (2013). A conceptual analysis of the knowledge of prospective mathematics teachers about degree and radian. *World Journal of Education, 3*(4), 1-9. https://doi.org/10.5430/wje.v3n4p1

Weber, K. (2008). Teaching trigonometric functions: Lessons learned from research. *Mathematics Teacher, 102*(2), 144-150. https://doi.org/10.5951/MT.102.2.0144

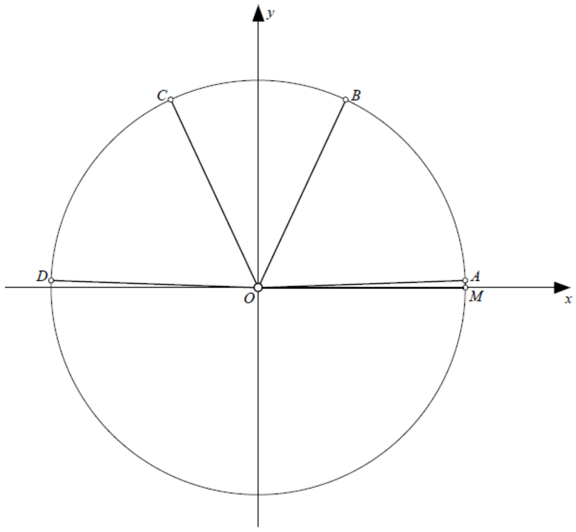
APPENDIX A

Test Items and Corresponding Standards

1. If angle α is given in degrees and $\alpha = 60°$, how much is α in radians?
   a. $\pi/3$
   b. $\pi/4$
   c. $\pi/6$
   d. $\pi/12$

Standards: The student converts degrees to radians, and vice versa.

2. Which of the angles in the following picture is an angle of 2 radians?



   a. $\sphericalangle MOA$
   b. $\sphericalangle MOB$
   c. $\sphericalangle MOC$
   d. $\sphericalangle MOD$

Standards: The student converts degrees to radians, and vice versa. The student connects basic trigonometric relations with the sketch and vice versa.

3. Radius of a unit circle is?
   a. 1
   b. $\frac{1}{2}$
   c. π
   d. $\frac{\pi}{2}$

Standards: The student connects basic trigonometric relations with the sketch and vice versa.

4. If $\alpha = \frac{\pi}{2}$, what are the values of sine and cosine functions?
   a. $\sin\alpha = 1,\ \cos\alpha = 0$
   b. $\sin\alpha = 0, \cos\alpha = 1$
   c. $\sin\alpha = 0, \cos\alpha$ is undefined
   d. $\sin\alpha$ is undefined, $\cos\alpha = 0$

Standards: The student connects and applies knowledge of trigonometry. The student connects basic trigonometric relations with the sketch and vice versa (calculates the values of the trig. functions).

5. If $\alpha = 0$, what are the values of tangent and cotangent functions?
   a. $\tan\alpha = 1, \cot\alpha = 0$
   b. $\tan\alpha = 0, \cot\alpha = 1$
   c. $\tan\alpha = 0, \cot\alpha$ is undefined
   d. $\tan\alpha$ is undefined, $\cot\alpha = 0$

Standards: The student connects and applies knowledge of trigonometry. The student connects basic trigonometric relations with the sketch and vice versa (calculates the values of the trig. functions).

6. For which of the following angles functions sine and cosine have the same value?
   a. 75°
   b. 150°
   c. 225°
   d. 300°

Standards: The student connects and applies knowledge of trigonometry.

7. Which trigonometric functions are positive for angles from the third quadrant?
   a. sine and tangent
   b. cosine and tangent
   c. sine and cotangent
   d. tangent and cotangent

Standards: The student connects and applies knowledge of trigonometry.

8. What is the expression $2\sin(-\alpha)\cos\beta + \sin\alpha\cos(-\beta)$ equal to?
   a. $-\sin\alpha\cos\beta$
   b. $\sin\alpha\cos\beta$
   c. $-3\sin\alpha\cos\beta$
   d. $3\sin\alpha\cos\beta$

Standards: The student converts mathematical models into new ones to solve a given problem.

9. What is the expression $2\tan(-\alpha)\cot\beta + \tan\alpha\cot(-\beta)$equal to?
   a. $-\tan\alpha\cot\beta$
   b. $\tan\alpha\cot\beta$
   c. $-3\tan\alpha\cot\beta$
   d. $3\tan\alpha\cot\beta$

Standards: The student converts mathematical models into new ones to solve a given problem.

10. If $\cot\alpha = 0.5$, for which of the following angles will the cotangent function have the same value, 0.5?
   a. $\alpha + \frac{\pi}{2}$
   b. $\alpha + \pi$
   c. $\alpha + \frac{3\pi}{2}$
   d. $\alpha - \frac{\pi}{2}$

Standards: The student connects and applies knowledge of trigonometry.

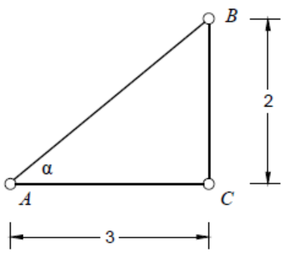
11. What is the expression $\frac{\cos\alpha}{1-\tan\alpha} + \frac{\sin\alpha}{\cot\alpha - 1}$ equal to?
   a. $\cos\alpha - \sin\alpha$
   b. $\cos\alpha + \sin\alpha$
   c. $\frac{1}{\cos\alpha - \sin\alpha}$
   d. $\frac{1}{\cos\alpha + \sin\alpha}$

Standards: The student converts mathematical models into new ones to solve a given problem.

12. What is $\sin 70°$ equal to?
   a. $\cos 20°$
   b. $\cos 70°$
   c. $\cos 110°$
   d. $\cos 290°$

Standards: The student applies triangle trigonometry in simple situations.

13. Calculate the value of $\tan\alpha$ for the right-angled triangle shown in the following picture:



a. $\frac{2}{(2^2+3^2)}$
b. $\frac{3}{(2^2+3^2)}$
c. $\frac{2}{3}$
d. $\frac{3}{2}$

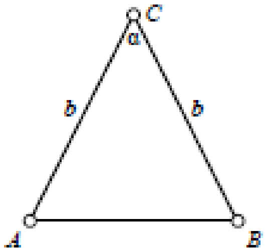
Standards: The student connects basic trigonometric relations with the sketch and vice versa (calculates the values of the trigonemetri functions). The student applies triangle trigonometry in simple situations.

14. If the length of one leg of a right-angled triangle is 1, and the angle between that leg and the hypotenuse is 60°, how long is the hypotenuse?

a. $\frac{\sqrt{3}}{2}$
b. 3
c. $\frac{\sqrt{2}}{2}$
d. 2

Standards: The student solves geometric problems using trigonometry.

15. Express the area of the isosceles triangle shown in the following picture, using the given length of side b and angle α:



a. $A = b^2 \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$
b. $A = \frac{1}{2}b^2 \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$
c. $A = b^2 \sin\alpha\cos\alpha$
d. $A = \frac{1}{2}b^2 \sin\alpha\cos\alpha$

Standards: The student solves geometric problems using trigonometry.

Correct Answers

1. a
2. c
3. a
4. a
5. c
6. c
7. d
8. a
9. c
10. b
11. c
12. a
13. c
14. d
15. a